Mathematics I (Part I) Honrs.

15|04|21

Partial and Total order relation.

Definitions:-

Partial order relation:- A relation R defined over a set A is said to be partial order relation if it is

(i) Reflexive: $\forall$ $a \in A$ such that aRa.

(ii) Antisymmetric: $\forall$ $a, b \in A$ such that $aRb, bRa \Rightarrow a = b$.

(iii) ~~Tositive~~ Transitive: $\forall$ $a, b, c \in A$ such that $aRb, bRc \Rightarrow aRc$

i.e. Let N = the set of all positive integers $N = \{1, 2, 3, 4, 5, 6, \ldots\}$ for any $m, n \in N$.

Let $m \leq n$ means that m divides n.

Then $(N, \leq)$ is a partially ordered set.

Totally order relation:- A partially ordered set $(A, \leq)$ is said to be a totally ordered set if for every $a, b \in A$ either $a \leq b$ or $b \leq a$.

Here the relation $\leq$ in called a totally ordering relation.

i.e. Let $A = \{2, 6, 8, 24, 72\}$. Define relation '$\leq$' on A means that $a \leq b$ iff a divides b.

Then the relation '$\leq$' in a totally ordered relation.

Theorem:- Distinguish between Partially and totally ordered sets by constructing an example of a partially ordered which is not totally ordered.

Proof:- From definition of partially and totally ordered set we see that every totally ordered set is partially ordered but converse is not necessarily true. Which is shown by following example.

let = $\{1,2,3,---\}$, For any $a,b \in N$.

let $a \leq b$ means that a divides b then $(N, \leq)$ is a partially ordered Set but it is not totally ordered.

For $(N, \leq)$ is a partially ordered set ~~but it is~~ ~~not totally ordered~~. We note that

(i) $a \leq a$ for all $a \in N$ since a divide a.

(ii) $a \leq b$ and $b \leq a \Rightarrow a = b \; (\forall \; a,b \in N)$

i.e. if a divides b and b divides a then $a = b$.

(iii) $a \leq b$ and $b \leq c \Rightarrow a \leq c \; (\forall \; a,b,c \in N)$

i.e. if a divides b and b divides c then We must have a divides c.

Here $(N, \leq)$ is a partially ordered set.

Now consider $3, 7 \in N$.

Here 3 does not divide 7 or 7 does not divide 3.

So $(N, \leq)$ is not a total order relation on N.

Thus $(N, \leq)$ is a partially ordered Set but not totally ordered.